

Introduction to Radar Imaging

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Outline

- Mathematical model
- Image formation
 - time domain viewpoint
 - frequency domain viewpoint (for small scenes)
- Approximating targets by point clouds
- SAR interferometry

Mathematical Model

Maxwell's equations \rightarrow scalar wave equation

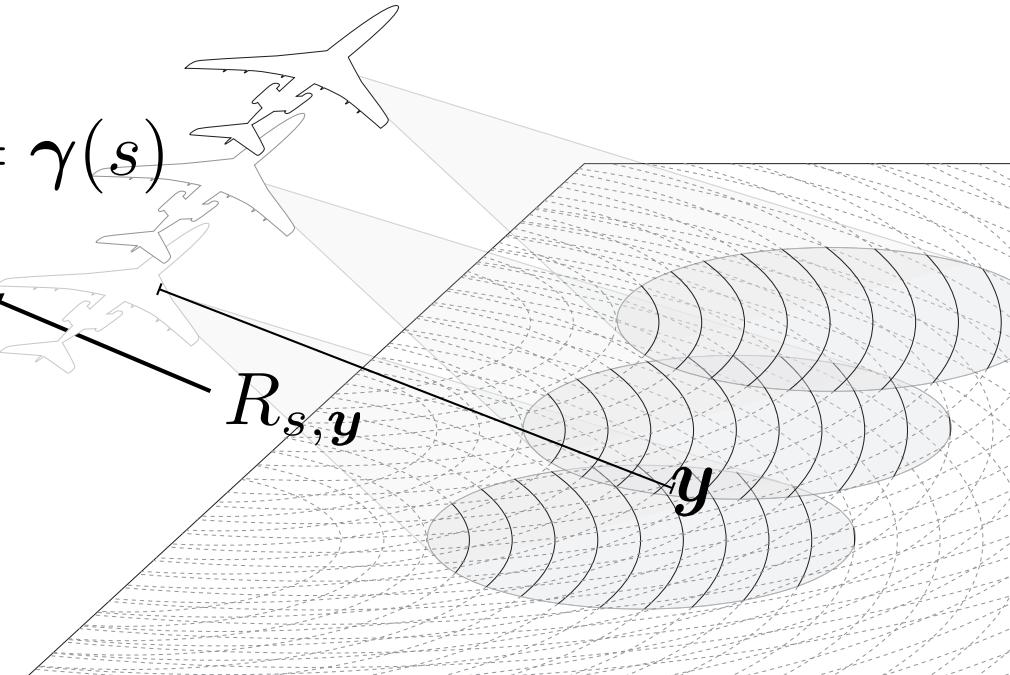
Green's function \downarrow + Born approximation

$$p(t, \mathbf{x}_r; \mathbf{x}_s) \propto \int \rho(\mathbf{y}) \frac{f''[t - \tau(\mathbf{y}, \mathbf{x}_s) - \tau(\mathbf{y}, \mathbf{x}_r)]}{|\mathbf{y} - \mathbf{x}_s| |\mathbf{y} - \mathbf{x}_r|} d\mathbf{y}$$

standard (monostatic) SAR:

$$\mathbf{x}_r = \mathbf{x}_s = \gamma(s)$$

$$\tau(\mathbf{y}, \mathbf{x}) = \frac{|\mathbf{y} - \mathbf{x}|}{c_0} = \frac{|\gamma(s) - \mathbf{y}|}{c_0}$$



$$p(t, s) \propto \int \rho(\mathbf{y}) \frac{f''(t - 2R_{s,\mathbf{y}}/c_0)}{R_{s,\mathbf{y}}^2} d\mathbf{y}$$

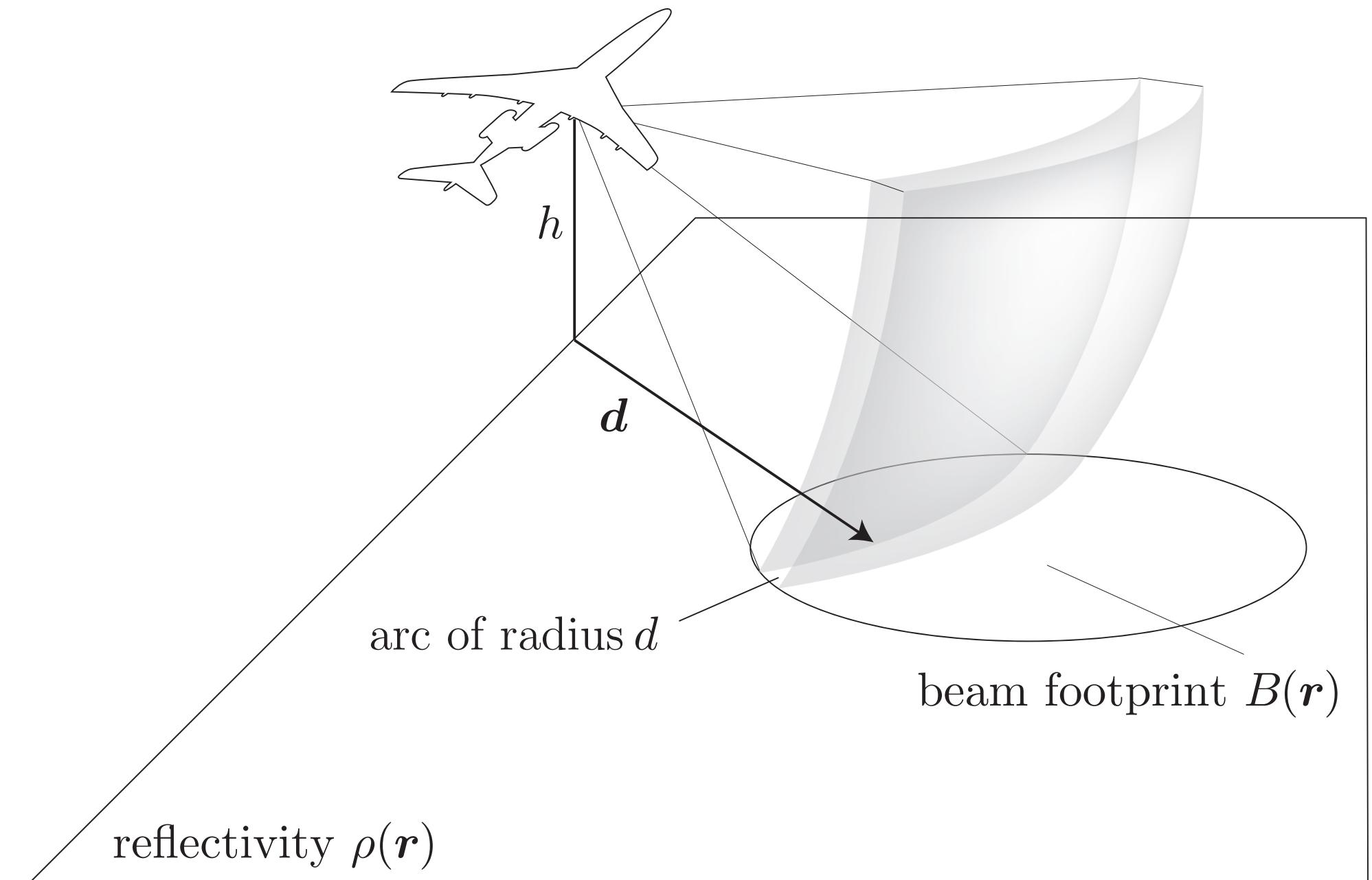


Image formation

$$I(\mathbf{y}) = \sum_{\mathbf{x}_r, \mathbf{x}_s} \text{data}(t = \tau(\mathbf{y}, \mathbf{x}_s) + \tau(\mathbf{y}, \mathbf{x}_r), \mathbf{x}_r; \mathbf{x}_s)$$

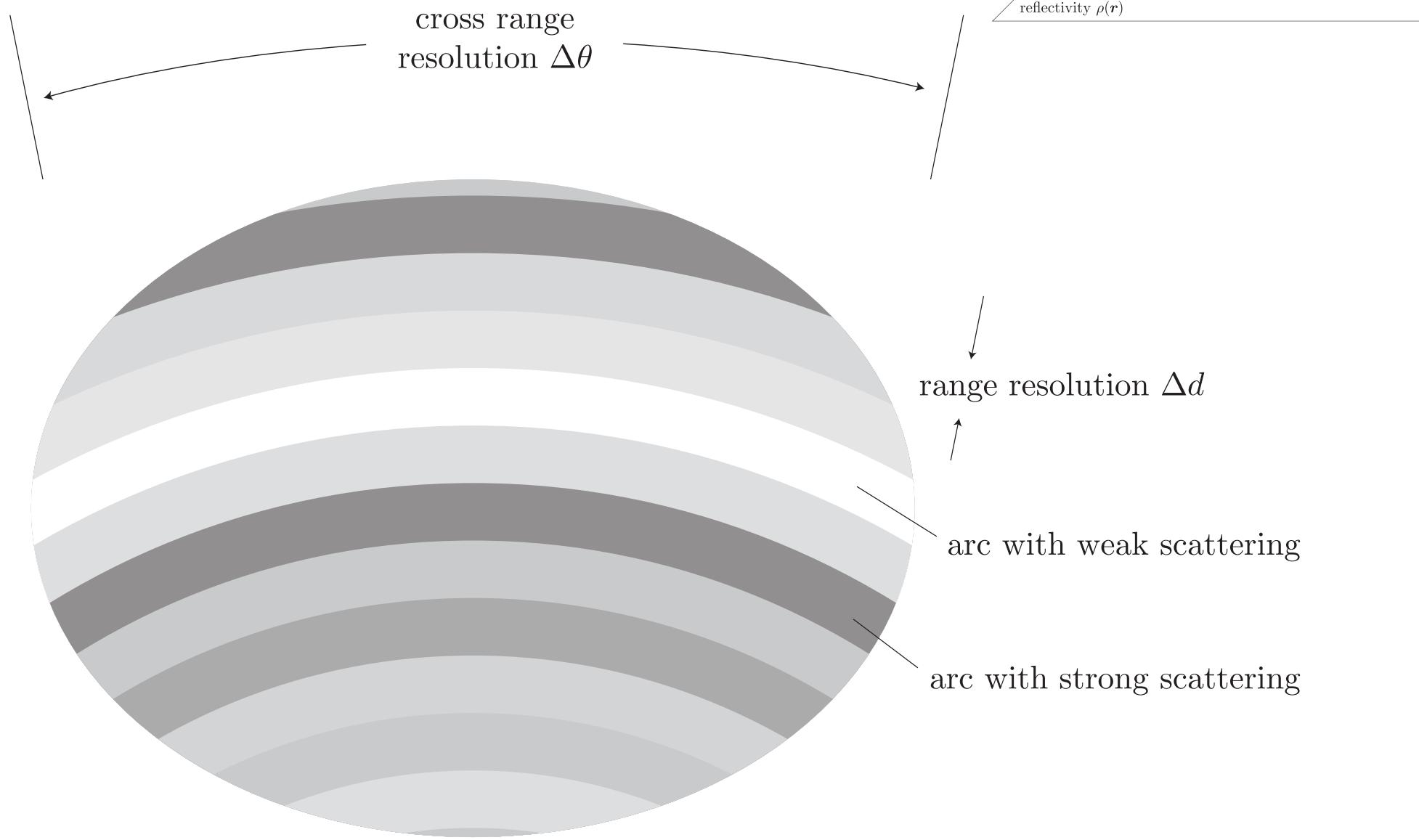
array imaging

$$= \sum_s \text{data}(t = 2R_{s,\mathbf{y}}/c_0, \gamma(s))$$

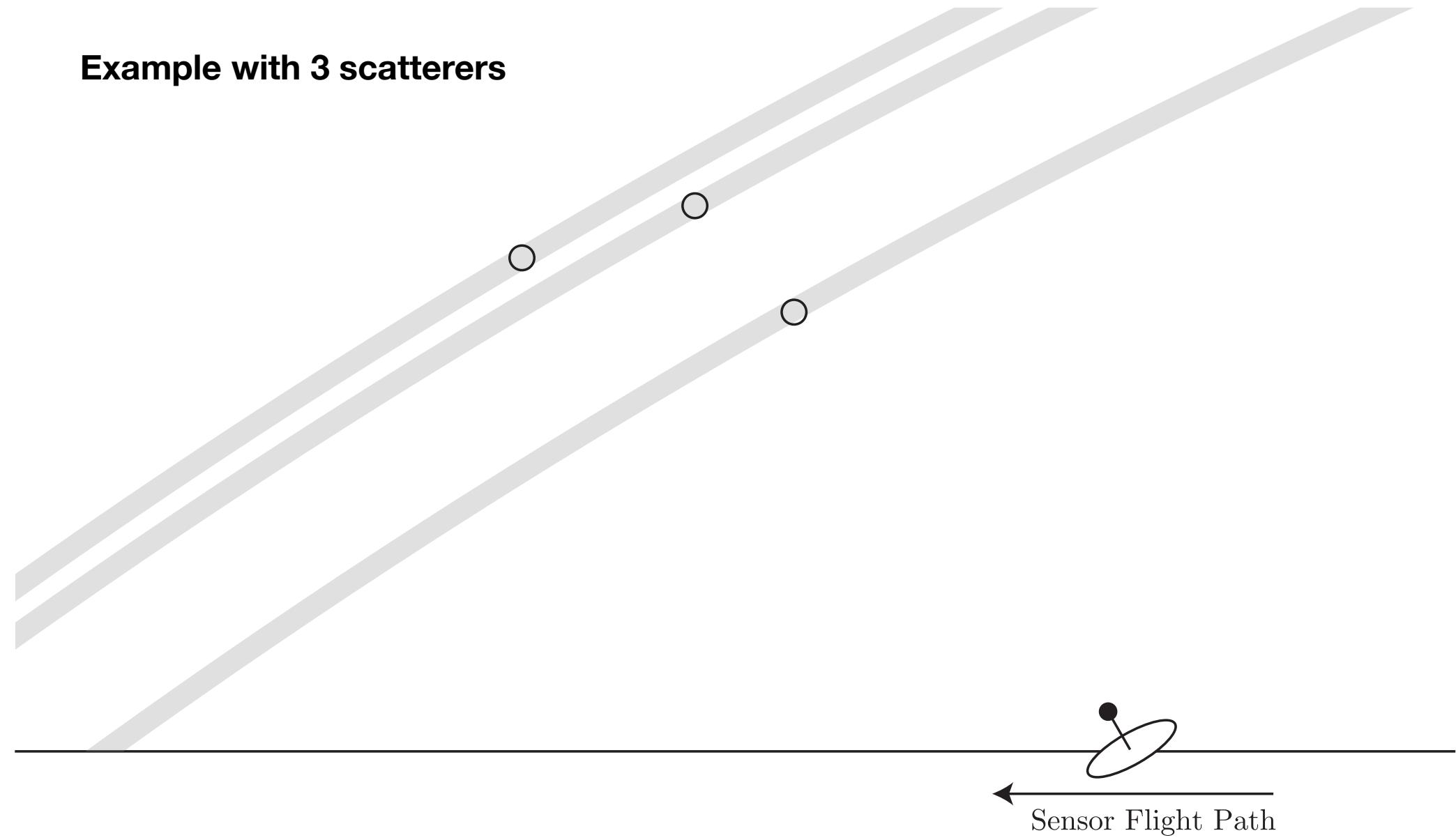
standard (monostatic) SAR

why does this work?

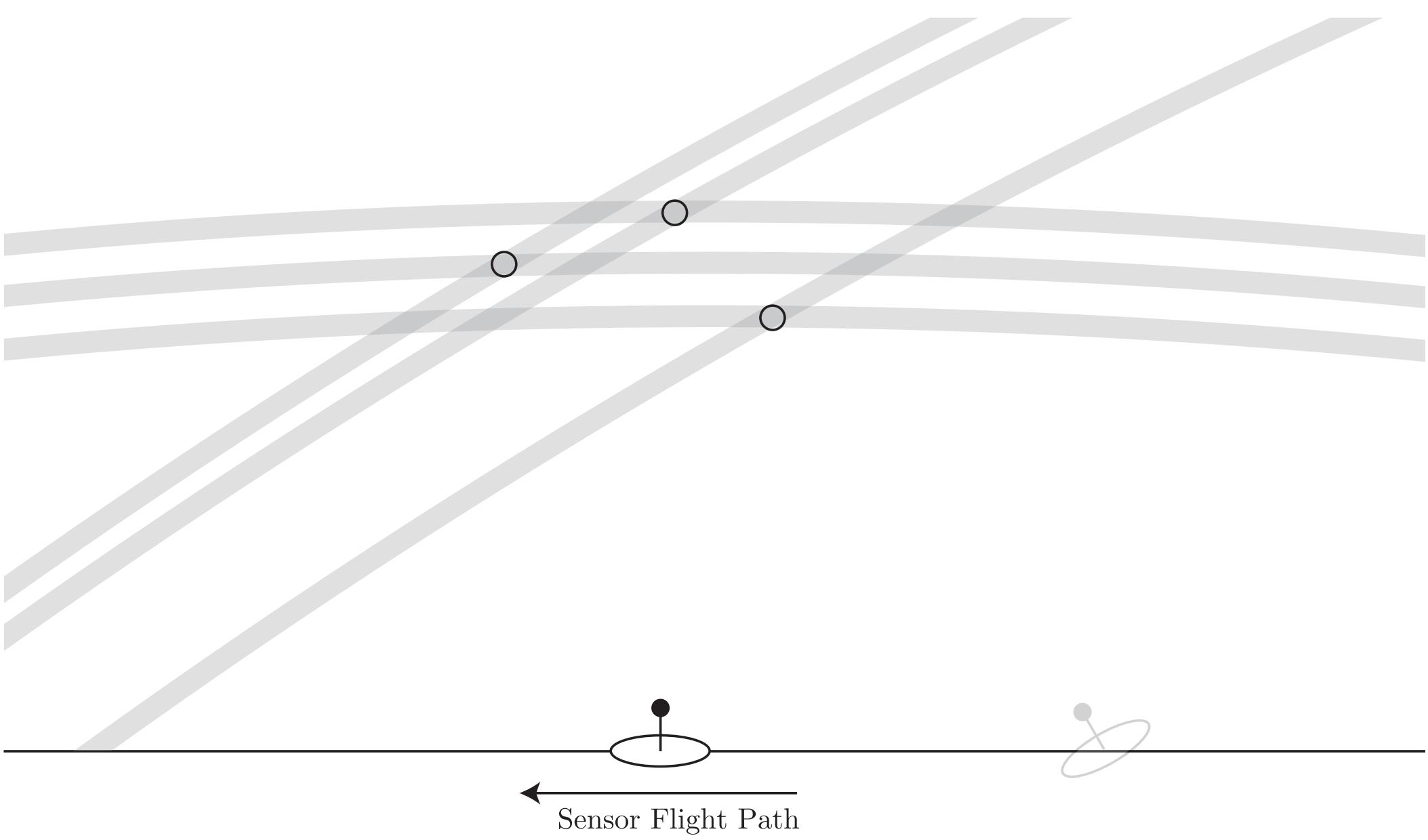
Imaging from a single viewing position



Example with 3 scatterers



Imaging from a single view



Imaging from two views

The diagram illustrates the principle of synthetic aperture imaging. A horizontal line represents the "Sensor Flight Path". Three points along this path are shown, each with a small circle and an elliptical sensor icon. From each sensor, several parallel grey lines radiate outwards at an angle, representing the "synthetic aperture". These lines converge towards three points in the upper portion of the diagram, each marked with a small circle. The text "Imaging from three views" is centered between the two middle sensor positions.

Imaging from three views

Sensor Flight Path

synthetic aperture

Frequency domain viewpoint

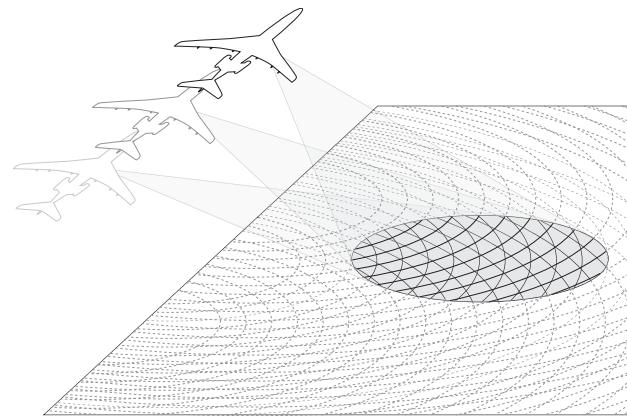
time-domain model

$$p(t, s) \propto \int \rho(\mathbf{y}) \frac{f''(t - 2R_{s,\mathbf{y}}/c_0)}{R_{s,\mathbf{y}}^2} d\mathbf{y}$$

Fourier transform in t

$$\downarrow \quad \int \dots e^{-i\omega t} dt$$

$$P(\omega, s) \propto \int \rho(\mathbf{y}) \frac{\omega^2 F(\omega) e^{-2i\omega R_{s,\mathbf{y}}/c_0}}{R_{s,\mathbf{y}}^2} d\mathbf{y}$$



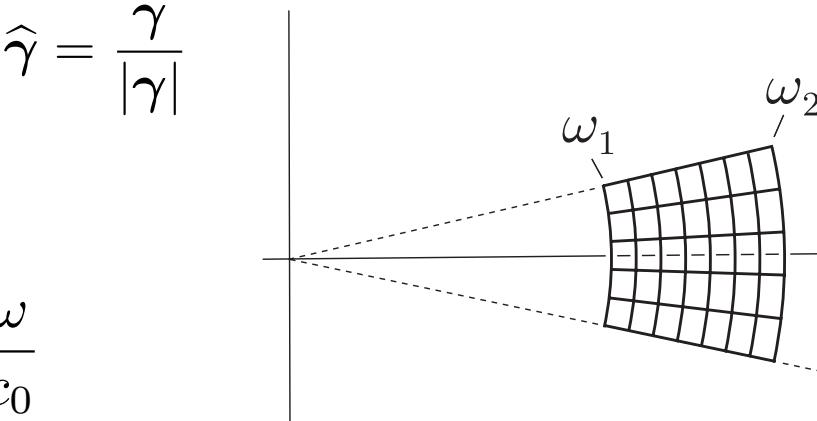
far-field approximation

$$R_{s,\mathbf{y}} = |\gamma(s) - \mathbf{y}| \approx |\gamma| - \hat{\gamma} \cdot \mathbf{y} + \dots \quad |\gamma| \gg |\mathbf{y}|$$

$$P(\omega, s) \propto \int \rho(\mathbf{y}) e^{-2ik\hat{\gamma}(s) \cdot \mathbf{y}} d\mathbf{y}$$

to form image, invert Fourier transform!

$$k = \frac{\omega}{c_0}$$

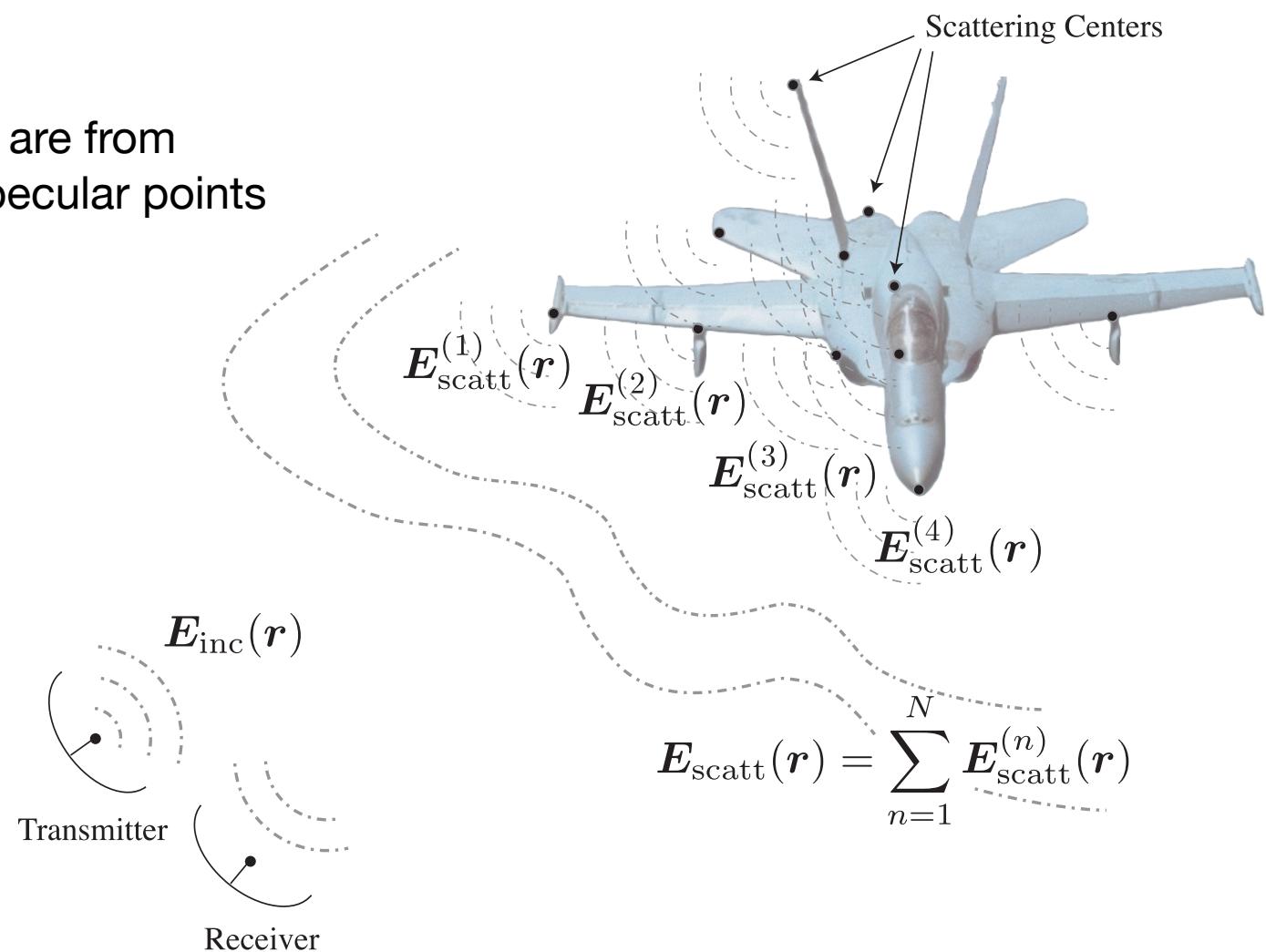


Approximating targets by point clouds

$$P(\omega, s) \propto \int \rho(\mathbf{y}) e^{-2ik\hat{\gamma} \cdot \mathbf{y}} d\mathbf{y}$$

k large \rightarrow use geometrical optics

main contributions are from corners, edges, and specular points



Interferometry

$$p(t, s) \propto \int \rho(\mathbf{y}) \frac{f''(t - 2R_{s,\mathbf{y}}/c_0)}{R_{s,\mathbf{y}}^2} d\mathbf{y}$$

narrowband

$$f(t) = a(t)e^{i\omega_0 t}$$

a = slowly varying (complex) amplitude

$$p(t, s) \propto \int \rho(\mathbf{y}) e^{i\omega_0(t - 2R_{s,\mathbf{y}}/c_0)} \frac{a(t - 2R_{s,\mathbf{y}}/c_0)}{R_{s,\mathbf{y}}^2} d\mathbf{y}$$

scattering takes place on surface

$$\rho(\mathbf{y}) = \tilde{\rho}(y_1, y_2)\delta(y_3 - h(y_1, y_2))$$

$$\mathbf{y} = \mathbf{y}_T + h(\mathbf{y}_T)\hat{\mathbf{e}}_3 \quad \mathbf{y}_T = (y_1, y_2, 0)$$

$$R_{s,\mathbf{y}} = |\mathbf{y}_T + h\hat{\mathbf{e}}_3 - \gamma| = \underbrace{|\mathbf{y}_T - \gamma|}_{R_{s,\mathbf{y}_T}} + \underbrace{h(\mathbf{y}_T)\hat{\mathbf{e}}_3 \cdot \widehat{\mathbf{y}_T - \gamma}}_{d(\mathbf{y}_T)}$$

$$p(t, s) \approx \int \left[\tilde{\rho}(\mathbf{y}_T) e^{2ik_0 d(\mathbf{y}_T)} \right] e^{i\omega_0(t - R_{s,\mathbf{y}_T}/c_0)} \frac{a(t - R_{s,\mathbf{y}_T}/c_0)}{R_{s,\mathbf{y}_T}^2} d\mathbf{y}_T$$

target phase encodes height information!